The Temporal Rich Club Phenomenon

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Identifying the hidden organizational principles and relevant structures of networks representing physical systems, ranging from the cellular scale to that of networks of social interactions, is fundamental for understanding their properties. In temporal networks one must further investigate the dynamic nature of such structures to uncover the key mechanisms of the time evolution of these systems and the processes unfolding on them.

In static networks, the "rich-club" phenomenon, i.e., the tendency of well-connected nodes to also connect to each other, has been extensively investigated. However, a well-connected structure in a static network might correspond in temporal networks to links that are never simultaneously present. Therefore, we propose here a novel measure to investigate the simultaneity and stability over time of such structures in temporal networks: the "*temporal rich club*". Specifically, we consider a temporal network in discrete time on a time interval $[0, T]$, and we denote by $G = (V, E)$ the corresponding aggregated graph. Our aim is to quantify whether the $N_{>k}$ nodes with degree larger than k in G (the sub-graph $S_{>k}$) are more connected *simultaneously* than by chance. Performing such analysis on a sub-network $S_{>k}$ with a large value of k allows to investigate the stability over time of the ties connecting the nodes with high centrality in the aggregated graph, i.e., whether a "*temporal rich club phenomenon*" is present. To this aim, we define $\epsilon_{>k}(t, \Delta)$ as the number of ties between nodes of $S_{>k}$ that remain stable over the time interval $[t, t + \Delta]$, normalized by the maximal density $|S_{>k}|(|S_{>k}| - 1)/2$, and we measure the maximal observed cohesion $M(k, \Delta)$ within $S_{>k}$ that remains stable during an interval of length Δ:

$$M(k, \Delta) \equiv \max_t \epsilon_{>k}(t, \Delta)$$

We test our novel measure on a range of temporal networks, ranging from temporal networks of face-to-face interactions in various contexts, to air transportation temporal networks and to brain functional connectivity networks.
For a given temporal network, we plot $M(k, \Delta)$ vs. k and Δ and compare it with a reshuffled version of the temporal graph (Figure 1.a-b). Figure 1.A shows that higher values of cohesion are observed, in face-to-face contact data, for $\Delta = 1$, even at low values of k, and that the cohesion increases with k. When the time-interval Δ increases, the temporal cohesion of sub-networks $S_{>k}$ drops rapidly for low values of k. For sub-networks composed by nodes of higher k, the cohesion decreases more slowly. We test the significance of such behaviour by performing the same analysis on a randomized version of the dataset (Figure 1.b): the randomization preserves the statistical properties of the aggregated graph G and the activity timeline of the temporal network, while randomizing the instantaneous edges. The comparison of original and reshuffled data highlights that the maximal cohesion $M(k, \Delta)$ reflects purely temporal structures. The dynamic evolution of such structures can be captured by studying the instantaneous cohesion $\epsilon_{>k}(t, \Delta)$ for different values of k and Δ (Figure 1.c): in this example, by plotting the cohesion of the sub-network $S_{>k}$ with $k = 88$, we find how this group of students is composed of children belonging to different classes, as the moments of highest instantaneous cohesion correspond to recess or lunchtime.

The analysis of temporal rich clubs can therefore help shed light on the existence and stability of *simultaneous* connections between the most connected nodes of a temporal network, and on their relation with specific moments of interest in the evolution of a temporal network.

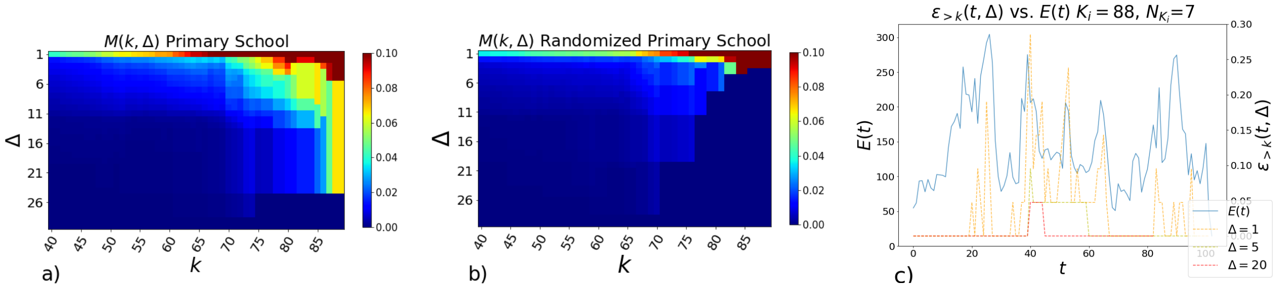

Figure 1: a) $k - \Delta$ diagram of the cohesion $M(k, \Delta)$ of a temporal network describing contacts between children in a primary school (www.sociopatterns.org); b) $M(k, \Delta)$ diagram for the same values of k and Δ as in the previous plot; c) Instantaneous values of the cohesion $\epsilon_{>k}(t, \Delta)$ of $S_{>k}$ for various values of the temporal resolution Δ, together with the instantaneous number of edges of the network $E(t)$ (in blue).