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Diffusion approximation of a network model of meme popularity

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The competition-induced criticality (CIC) model [1] reproduces meme popularity dynamics through an agent-based model, which is in turn mathematically tractable with a branching processes description. The model has its name because its dynamics lead to critical phenomena, namely the scaling recovered from systems with self-organised criticality. Its information cascades are reminiscent, for example, of avalanche scaling under sandpile models [2].

In stochastic terms, the way memes occupy screens and propagate through sharing is a discrete-time jump process. In this work, we offer a way to formulate dynamics instead as a continuous diffusion process, by aggregating and averaging random jump events into a continuous distribution with the central limit theorem.

The approximation is derived on a simpler version of the CIC model. We consider a directed network of followership relations, directed connections between users through which memes are transmitted. Each user has a screen that displays one meme at a time. At each discrete timestep, a randomly selected user is "activated", where it either sends the meme currently occupying their screen to the screens of all other users following them or *innovates*, introducing a new meme to its own screen, and also overwriting those of its neighbours.

The key step in our approximation is to consider a time window that aggregates many activation events. During this window, the change in our quantity of interest, the fraction of screens s occupied by a given meme, can then be estimated through the classic and generalised central limit theorems. A stochastic difference equation governing s follows, which numerically approximates the evolution of s over time. We validate these approximated trajectories computationally by examining meme popularity distributions, a proxy of the screen fraction s.

A highlight of our diffusion approximation is that we can reproduce the scaling meme popularity exhibits [3] with significantly less computational cost. Panel (a) of Figure 1 corresponds to this reproduction with a power-law out-degree network, which we account for using the so-called generalised central limit theorem. To do this, we resort in turn to stable distribution theory. Furthermore, we obtain an analytical expression for the time evolution of $\langle s \rangle$ from the associated Fokker-Planck equation and compare it with the CIC agent-based network stimulation in panel (b).

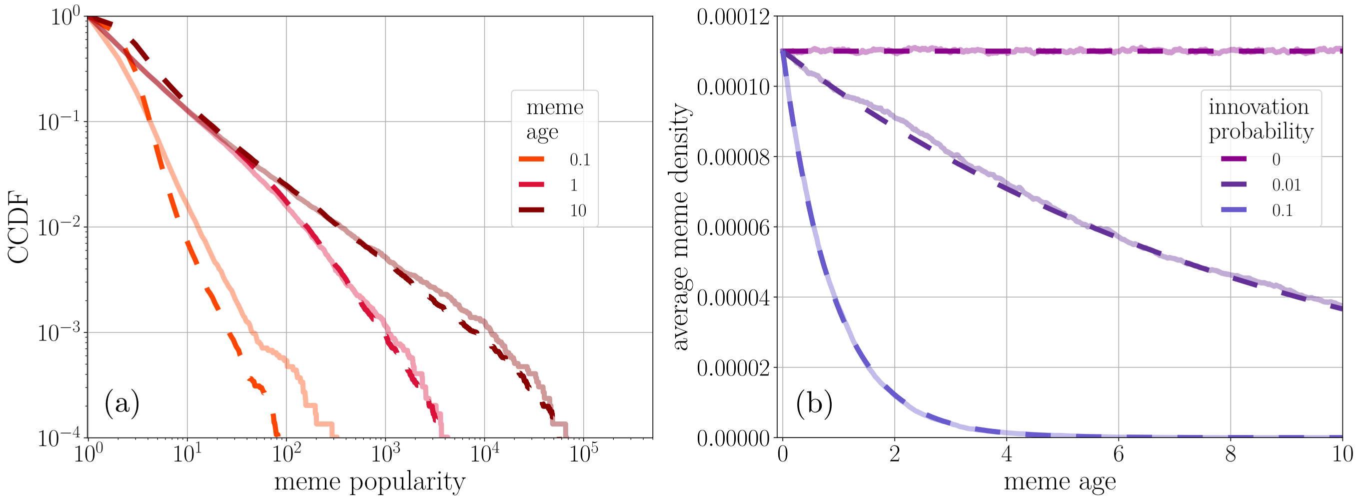

FIG. 1. Comparison of the diffusion approximation (dashed line) with the agent-based network simulation (solid line) of the competition-induced criticality (CIC) model. In (a), meme popularities in a scale-free network are compared with innovation probability 0.01; in (b), time evolution of the average meme density s, up to age 10, for different values of innovation probability, in a regular-degree network.

Our diffusion approximation [4] of a meme popularity model sheds light on agent-based spreading processes on networks, by way of vastly simplifying and accelerating numerical experiments while retaining accuracy. The resulting method leverages independent trajectories that do not depend on network size, a constraint in the corresponding network simulations, where constant updating of screens is required. Moreover, it enables investigation of novel aspects of the model dynamics, such as the temporal evolution of the moments of the meme screen density s. This new perspective paves the way for future work in which slight modifications can be made to look at different meme spreading dynamics, such as when memes attract user attention in heterogeneous ways (i.e., some are more likely to spread than others), sometimes referred to as fitness models.

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