Boundary effects in diffusion of new products

Gadi Fibich\(^1\), Tomer Levin\(^1\), and Kenneth Gillingham\(^2\)

\(^1\) School of Mathematical Sciences, Tel Aviv University, Tel Aviv, Israel, fibich@tau.ac.il, levintmr@gmail.com.
\(^2\) Yale University, New Haven, CT 06033, USA, kenneth.gillingham@yale.edu

The discrete Bass model on 2D Cartesian networks describes the diffusion of new products that spread primarily by spatial peer effects, such as residential photovoltaic solar systems. This model was previously analyzed for infinite 2D networks and for finite 2D networks with periodic boundary conditions. In both cases, translation invariance leads to a huge simplification in the analysis. In practice, residential units belong to a finite 2D network (e.g., a town) with external and possibly internal boundaries. The lack of translation invariance makes the analysis considerably more complex. Here, we analyze, apparently for the first time, the quantitative effect of boundaries in the discrete Bass 2D model, and show that:

1. Nodes near the town boundary are less likely to adopt than centrally-located ones.
2. This boundary effect is local, and decays exponentially fast with distance.
3. At the aggregate level, the reduction of the adoption level by the boundary scales as \(1/\sqrt{M}\), where \(M\) is the number of nodes.

Our theoretical analysis is supported by the first empirical evidence for the effects of boundaries on the diffusion of residential rooftop solar in Connecticut, which shows that internal and external boundaries reduce the adoption probability of near-boundary units by roughly 30\(\%\) − \(50\%\).

Fig. 1. Top: Cluster expansion above a horizontal barrier. Bottom: Same, when barrier has a hole.

Fig. 2. (A) Map of Connecticut with all Solarize municipalities (2012-2019). (B) Examples of water boundaries (on left) and boundaries crossing contiguous neighborhoods (on right).