BOUNDED-CONFIDENCE MODELS WITH ADAPTIVE CONFIDENCE BOUNDS*

GRACE J. LI†, JIAJIE LUO†, AND MASON A. PORTER‡

Introduction. People's opinions change with time as a result of interactions. In particular, individuals are often influenced by others with similar opinions. This phenomenon is captured by *bounded-confidence models* (BCMs), a family of opinion models with continuous-valued opinions. In a BCM, individuals (which are represented by nodes in a network) are influenced only by neighbors whose opinions are within their confidence bound. We introduce two discrete-time BCMs — a synchronous (in which all node pairs interact in each time step) and an asynchronous (in which only one pair of nodes interacts in each time step) — with confidence bounds that are heterogeneous and adaptive. We analytically and numerically explore the limiting behavior of our models, including the confidence-bound dynamics, the formation of clusters of nodes with similar opinions, and the time-evolution of the "effective graph," which is a time-dependent subgraph of the network with edges between nodes that can currently influence each other.

Our Models. We suppose that opinions take values in $[0, 1]$. Our asynchronous and synchronous models are based on the Deffuant–Weisbuch (DW) [1] and Hegselmann–Krause (HK) models [2], respectively. We thus refer to them as adaptive-confidence DW and HK models. The opinion update rules in our adaptive-confidence DW and HK models are the same as those in the original DW and HK models. The original models have time-independent and homogeneous confidence bounds, whereas our adaptive-confidence models update the confidence bounds of each pair of nodes. Each edge (i, j) has its own time-dependent confidence bound $c_{ij}(t)$. When nodes i and j interact, c_{ij} increases if their interaction results in an opinion change (which we interpret as a positive interaction) and decreases if it does not.

Summary of our Current Results. We analytically explore the limiting behavior of the confidence bounds. We show that for each pair of nodes i and j, the limit $\lim_{t\to\infty} c_{ij}(t)$ of their confidence bounds exists and is either 0 or 1. We also give results for the limiting behavior of the effective graphs in our models. We show numerically that our models achieve consensus for a wider range of parameters for complete graphs than the original DW and HK models. In our ongoing work, we are extending our study to other graphs, including Facebook networks and Erdős–Rényi models.

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†Department of Mathematics, University of California, Los Angeles, CA, USA (graceli@math.ucla.edu, jerryluo8@math.ucla.edu). These authors contributed equally

‡Department of Mathematics, University of California, Los Angeles, CA, USA; Santa Fe Institute, Santa Fe, NM, USA (mason@math.ucla.edu).