

# ESTIMATION OF UNKNOWN PAYOFFS PARAMETERS IN LARGE NETWORK GAMES

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## Abstract

Systems involving very large numbers of autonomous agents making strategic decisions and influencing each other over a network structure are becoming ubiquitous in applications such as power and traffic networks, targeted marketing and opinion dynamics. Studying how agents make decisions in these complex environments is essential for the successful design of interventions aimed at improving welfare or system performances. To this end, game theoretical principles are typically used to model agents' decisions via payoff maximization, resulting in the concept of Nash equilibrium (i.e., a set of actions in which no agent has interest for unilateral deviations) as solution outcome. While the parametric form of agents' payoff functions might be known, in most applications the parameters themselves are not. For example, in games capturing agents' decisions under peer pressure, the strength of neighbors' peer effect on an individual's marginal return might not be known for a particular network instance [1]. Whether a central planner can estimate the unknown parameters from observations of agents' actions at equilibrium in these settings affects its ability to design interventions to improve welfare or system efficiency.

Most recent work on payoff parameter estimation in network games either assume exact knowledge of the network (which might not be available for applications involving a large number of agents) or assume a specific structure on the payoff function (e.g., linear quadratic payoff). The key objective of this work is to overcome these limitations by developing a novel parameter estimation procedure that: i) relies on statistic instead of exact information about network interactions and ii) can be applied for parameter estimation in generic network games, beyond models with linear quadratic payoff functions. To obtain such a result, we build on the framework of graphon games [3], which are games played over a continuum of agents that interact heterogeneously according to a graphon. Building on an interpretation of graphons as random network

models (which generalizes for example Erdős-Rényi and stochastic block models) [2], it was previously shown that equilibria of network games in which the network of interactions is sampled from the graphon (termed sampled network games) converge, in the limit of large populations, to the equilibrium of the corresponding graphon game [3]. Equilibria of graphon games can thus be seen as an approximation of strategic behavior in large network games, computed by using only information about the random network model. Based on this result, given an observation of the equilibrium of a sampled network game with unknown parameters, the proposed approach consists of selecting as estimator the parameter for which the equilibrium of the corresponding graphon game is the closest to the observed equilibrium.

As first contribution, we demonstrate that this estimator is locally unique and asymptotically consistent if the parameter satisfies an identifiability assumption capturing games in which equilibria that are close are generated by parameters which are also close. Finding the parameter for which the graphon game equilibrium is the closest to the observed equilibrium requires the solution of an optimization problem. As second contribution, we show that the cost function of such optimization problem is smooth and locally strictly convex around the true parameter. Moreover, under the identifiability assumption above, we also show that the optimization problem admits a unique global optimizer, thus guaranteeing that the proposed estimator is unique and well-defined. Finally, we present several examples of identifiable parameters in different classes of network games.

## References

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