The multiplex decomposition: An analytic framework for multilayer dynamical networks.

Rico Berner *, Volker Mehrmann †, Eckehard Schöll ‡, and Serhiy Yanchuk §

Complex networks are well-established models in science and technology, with a wide range of applications from physics, chemistry, biology, neuroscience, as well as engineering and socio-economic systems. Much work has been devoted to understanding the statistical and topological properties of and the collective dynamics on complex connectivity structures. In this study, we focus on synchronization which is a particularly important type of collective dynamics playing an important role in the theory of complex dynamical systems, e.g. power grids or neural systems. One of the most powerful methodologies to study the synchronization on complex network structures is the master stability approach.

A recent focus in the field of complex dynamical networks are multilayer networks. A special case of multilayer networks are multiplex topologies, where each layer contains the same set of nodes and there are only pairwise connections between corresponding nodes from different layers. These structures possess remarkable and analytically accessible properties that have been widely studied and used to understand real-world networks. We consider the special class of multiplex networks where the adjacency matrices for each layer are simultaneously triagonalizable. For such networks, we derive the relation between the spectrum of the multiplex network and the eigenvalues of the individual layers. As an application, we propose a generalized master stability approach that allows for a simplified, low-dimensional description of the stability of synchronized solutions in multiplex networks. We illustrate our result with a duplex network of FitzHugh-Nagumo oscillators. In particular, we show how interlayer interaction can lead to stabilization or destabilization of the synchronous state. Finally, we give explicit conditions for the stability of synchronous solutions in duplex networks of linear diffusive systems.

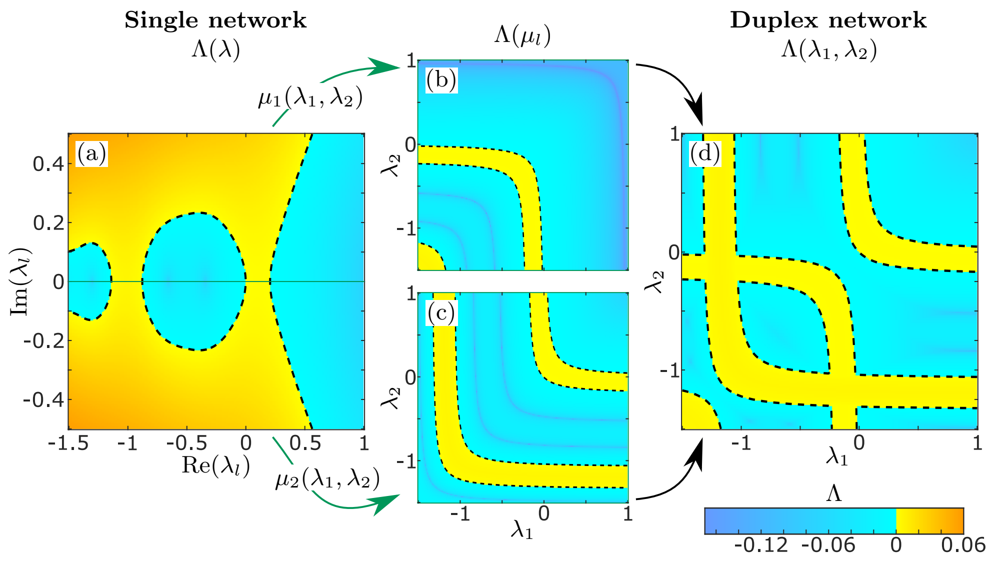

Figure 1. *Construction of the master stability function for a duplex network of coupled FitzHugh-Nagumo oscillators.*

*Department of Physics, Humboldt-Universität zu Berlin, Germany, (rico.berner@physik.hu-berlin.de).
†Institute of Mathematics, Technische Universität Berlin, Germany.
‡Institute of Theoretical Physics, Technische Universität Berlin, Germany.
§Potsdam Institute for Climate Impact Research, Germany.