Characterisation of large spatial motifs in random colouring processes

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In many real-world spatial complex systems, including urban environments and biological tissues, the units of the system are often associated with classes or labels **[1,2]**. In such systems, the emergence of non-trivial motifs and clusters of regions belonging to the same class are the norm rather than the exception.
The use of labels to represent physical quantities on the network representation of real spatial systems is tied up with the need to understand and quantify the presence of heterogeneity in the distribution of the variable of interest. The characterisation of spatial heterogeneity normally involves the comparison with a reference distribution observed in a null-model: this is crucial to assign a statistical significance to the measured quantities when a colouring process is implemented. In fact, in the case of spatial networks, the presence of spatial constraints could lead to the emergence of structures with sensible dimension **[3]**.
In this work, we characterise and explore spatial motifs generated by a simple random colouring process, called Uniform Random Colouring (URC): to each node of the network is assigned one of C colours at random, independently of any other node of the graph, by sampling it according to a pre-determined colour distribution. This process is the simplest stochastic process that preserves a given colour distribution but destroys correlations among colours, and is therefore the best candidate for a minimalist null-model of spatial heterogeneity and correlations.
We compare the distribution of the sizes of the motifs produced by URC in the case of Random Regular Graph (RRG), used as null-model, and a 2D lattice, where each node has degree k=4. We observe that, in the case of the lattice and small number of labels, the simple presence of spatial costrains generates, on average, larger motifs than in the case of RRG.
We derive a dynamical Random Growth Model (RGM) to reproduce the ensemble of configurations generated by URC and to obtain the same large motifs as in URC. We measure some fundamental and structural quantities of these structures, making a comparison with a well-known growth model, the Eden Growth Model (EGM) **[4]**, used as control.
Then, we show how the trajectories of random walks on such structures retain important information about their size and shape, and that the distribution of inter-class passage times can be used to characterise these patterns.
This study points out the presence of extended random structure when the space of colours of the colouring process is small, and iis related to the necessity of obtaining significant measures of real quantities on coloured spatial networks and adequate null models, commensurate to the specific spatial network in which the colouring process is implemented.

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