Embodied Hypergraphs: analyzing interactions in time-dependent systems  
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Complex network analysis is best suited for looking at interactions across many constituent parts. In cases where multiple timepoints need to be modeled, dynamical or multilevel approaches can often be useful. For systems with an inheritance property, however, these approaches are lacking. Embodied Hypergraphs offer a number of alternatives to this state of affairs. Embodying a directed network enables us to reconcile aspects of spatial and/or anatomical features in a number of biological systems. More generally, embodied hypergraphs focus on inheritance between layers of the network, analogous to the type of heritability found in biological and cultural evolution. The flow and unfolding of events over time, or generativity of a process, is uniquely captured by this class of network. Aside from the properties of spatial/anatomical, inheritance, and generativity, embodied networks are important for both orienting directed networks in the physical world. Embodied hypergraphs bridge the gap between complex network theory and real-world systems, while also providing a nested context to systems that are temporally variable and active at multiple spatial scales.

We can further define embodied networks by turning to examples from embryogenesis, neurodevelopment, and cultural evolutionary systems. In the first example, the formation of newly-born tissue types and organs are characterized as a discrete spatiotemporal process. The neurodevelopmental example models a small connectome in an embodied agent as it grows and initiates sensorimotor behavior. The cultural inheritance example enables the flexibility of cycles and recombination in a directed tree. For each example, we will walk through the properties of an embodied hypergraph. Hypernodes capture the variety of cells and or ideas in our networks, while heritability and cycles between hypernodes are characterized by the hyperedges. The objects within a hypernode are further characterized by categories, but perhaps more importantly, functional mappings between categories. Category theory allows us to define these hypercycles, which are essential for characterizing variety and recombinant processes as they unfold in the network. Finally, we will consider novel network statistics that might incorporate the multitude of special cases presented here. Overall, embodied hypergraphs are both of great theoretical interest and practical utility.