Generative Hypergraph Models and Spectral Embedding

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Many complex systems involve interactions between more than two agents. Hypergraphs capture these higher-order interactions through hyperedges that may link more than two nodes. We consider the problem of mapping nodes in a hypergraph into one or higher dimensional locations such that most interactions are short-range. This embedding is relevant to many follow up tasks, such as node reordering, node clustering, and visualization. We show that two spectral hypergraph embedding algorithms, which reveal linear and periodic structures respectively, are associated with a new class of hypergraph generative model [1]. The model assigns a probability to each hyperedge which decays with the sum of the squared pairwise distances between nodes in the hyperedge, therefore encouraging short-range connections. This random graph model allows us to quantify the relative presence of periodic and linear structures in the data through maximum likelihood, and provides a metric for hyperedge prediction. We demonstrate this approach on synthetic and real-world high-school contact hypergraphs [2, 3]. We perform prediction of triadic hyperedges(triangles) using the proposed linear model and show that it outperforms the existing average-score based method [4] on synthetic hypergraphs, and also on high school contact data when the amount of training data is limited.

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