DESIGNING ROBUST NETWORKS OF COUPLED PHASE-OSCILLATORS WITH APPLICATIONS TO THE HIGH VOLTAGE ELECTRIC GRID

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Summary

We propose a mathematical framework for designing robust networks of coupled phase-oscillators by leveraging a vulnerability measure that quantifies the impact of a perturbation at an individual phase-oscillator's natural frequency to the system's global synchronized frequencies. Given a network topology, the proposed framework finds an optimal allocation of edge weights that minimizes such vulnerability measures at nodes for where perturbations are expected to occur by solving a tractable semi-definite programming problem. We specify the mathematical model to high voltage electric grids.

Background and Motivation

We consider a weighted, connected, and undirected network, $G = (V, E)$, where V is a set of n nodes and E is a set of m edges. We denote $B \in \mathbb{R}^{n\times n}$ to be the weighted adjacency matrix specifying the edge weights of G such that $B_{ij} = b_{ij} \geq 0$ if $(i,j) \in E$ and $B_{ij} = 0$ if $(i,j) \notin E$. A network of coupled phase-oscillators is a network where each node $i \in V$ corresponds to an angle, $\theta_i \in [-\pi, \pi)$, that evolves according to the coupled dynamics, $\dot{\theta}_i = \omega_i - \sum_{j=1}^{n} B_{ij} \sin(\theta_j - \theta_i)$. The phase-oscillator's dynamics of node i is determined by its natural frequency, ω_i, and its coupling with other phase-oscillators determined by the network's edge weights, B.

This coupled phase-oscillator model has been utilized to describe high voltage electric grids [1]. In this application, it is necessary for phase-oscillators to maintain global synchronized frequencies, (i.e., $\dot{\theta}_i = \omega_0$ for all $i \in \{1, \ldots, n\}$). However, the integration of renewable energy poses a challenge to a grid's capacity to maintain synchronized frequencies because renewable generation is typically stochastic, resulting in small perturbations to the power injected into the system at nodes captured by the natural frequency, ω_i. Motivated by this application, a network of coupled phase-oscillators is defined as *robust* if the oscillators in the system *maintain synchronized frequencies throughout time despite small perturbations to a set of oscillators' natural frequencies.*

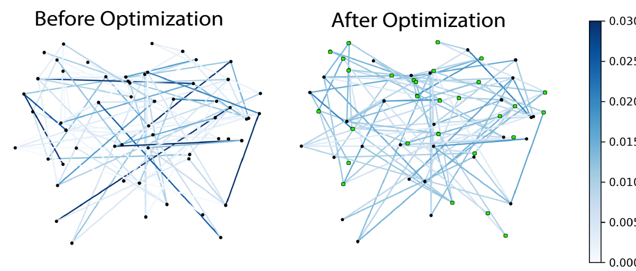

Figure 1: Here, V' consists of 29 nodes (indicated in green) where we wish to integrate renewable energy. Redistributing the edge weights as suggested by our framework lends to a 53.6% decrease in the sum of vulnerability measures at nodes in V'

The Design Problem

Building on [2] and by making a small angle difference approximation, we are able to quantify how much a small perturbation to a node k's natural frequency, ω_k, impacts the system's global synchronized frequencies with a *vulnerability measure* $\hat{\mathcal{M}}_k(b)$, which depends only on the vector of edge weights b. To increase the robustness of the network, we find an optimal weight allocation to minimize the worst case vulnerability measure at a given set of nodes, $V' \subseteq V$. Mathematically, we aim at finding $b^* \in \mathbb{R}^m$ such that $b^* = \arg\min_b \max_{k \in V'} \hat{\mathcal{M}}_k(b)$ subject to $b \geq 0$ and $\mathbf{1}^T b = 1$. We show that this is a convex optimization problem and solve it efficiently by reformulating it into a semi-definite programming problem. Figure 1 applies the optimization framework to an electric grid test case where $|V| = 57$.

References

[1] Florian Dörfler and Francesco Bullo. Exploring synchronization in complex oscillator networks. In *2012 IEEE 51st IEEE Conference on Decision and Control (CDC)*, pages 7157–7170. IEEE, 2012.

[2] Melvyn Tyloo, Laurent Pagnier, and Philippe Jacquod. The key player problem in complex oscillator networks and electric power grids: Resistance centralities identify local vulnerabilities. *Science advances*, 5(11):eaaw8359, 2019.