The Small-World Effect for Interferometer Networks

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Abstract

Complex network measures including clustering and path-length quantify well-defined aspects of complex networks, and they have been applied broadly to characterize networks arising in many applications including sociology, neuroscience, economics, and physics [1]. For example, networks with high clustering and small characteristic path length are designated as small world networks [2]. However, real-valued complex network measures are ill-suited for addressing physical problems with phase. To address this gap, we generalize clustering and path length measures to account for complex-valued edge weights. Using interferometer networks as a model for phased systems, we create a small-world interferometer model that uses an unweighted, directed small-world network as a skeleton with complex number edge weights to characterize the attenuation and phase shift of light as it passes between parts of the network. We computationally test these generalized measures on small-world interferometers, and we define a generalized small-world coefficient [3] that uses the generalized clustering and path length to account for the effect of interference on signal transfer. We show that interference is relevant to the small-world effect: the small-world coefficient computed with the generalized measures depends on the phase of edge weights in the network and ranges from slightly lower to several times higher than the traditional value of the small-world coefficient computed with only the traditional, real-valued measures. These results reveal that large phase angles cause the breakdown of the small-world effect, but small phase angles amplify the small world effect greatly. This demonstrates the need for accounting for phase when applying network measures to interfering systems.

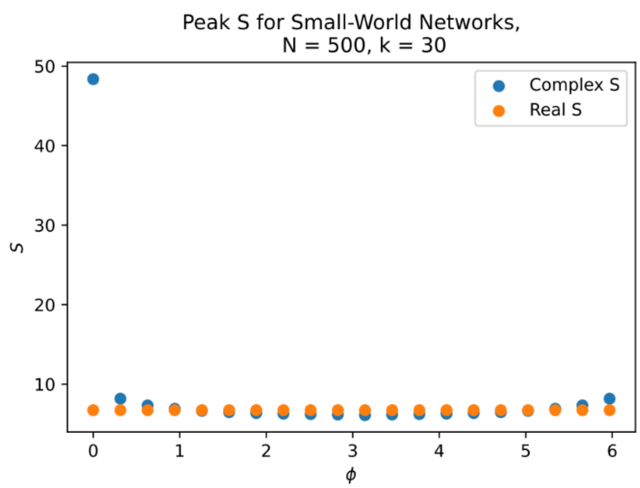

Figure: Complex-valued small-world coefficient S varies with phase, φ. Real-valued small world coefficient S is shown for reference and sits at a constant value of 1. This plot shows the results for one representative case: networks with 500 nodes and 30 connections per node.

References

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