

Sandpile cascades on oscillator networks: the BTW model meets Kuramoto

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Cascading failures abound in complex systems and the BTW sandpile model provides a theoretical underpinning for their analysis. Yet, it does not account for the possibility of nodes having oscillatory dynamics such as in power grids and brain networks. Here we consider a network of Kuramoto oscillators upon which the BTW model is unfolding, enabling us to study how the feedback between the oscillatory and cascading dynamics can lead to new emergent behaviors. We assume that the more out-of-sync a node is with its neighbors the more vulnerable it is and lower its load-carrying capacity accordingly. And when a node topples and sheds load, its oscillatory phase is reset at random. This leads to novel cyclic behavior at an emergent, long timescale. The system spends the bulk of its time in a synchronized state where load builds up with minimal cascades. Yet, eventually the system reaches a tipping point where a large cascade triggers a “cascade of larger cascades,” which can be classified as a dragon king event. The system then undergoes a short transient back to the synchronous, build-up phase. The coupling between capacity and synchronization gives rise to endogenous cascade seeds in addition to the standard exogenous ones, and we show their respective roles. We establish the phenomena from numerical studies and develop the accompanying mean-field theory to locate the tipping point, calculate the load in the system, determine the frequency of the long-time oscillations and find the distribution of cascade sizes during the build-up phase.

The coupled BTW Kuramoto model is associated with an underlying discontinuous phase transition with hysteresis. Such systems have been studied under the name of “self-organized bistability” and are known to produce dragon kings. However this is not the only class of dragon king capable models. We study a second class of dragon king models that are associated with continuous transitions and no hysteresis.

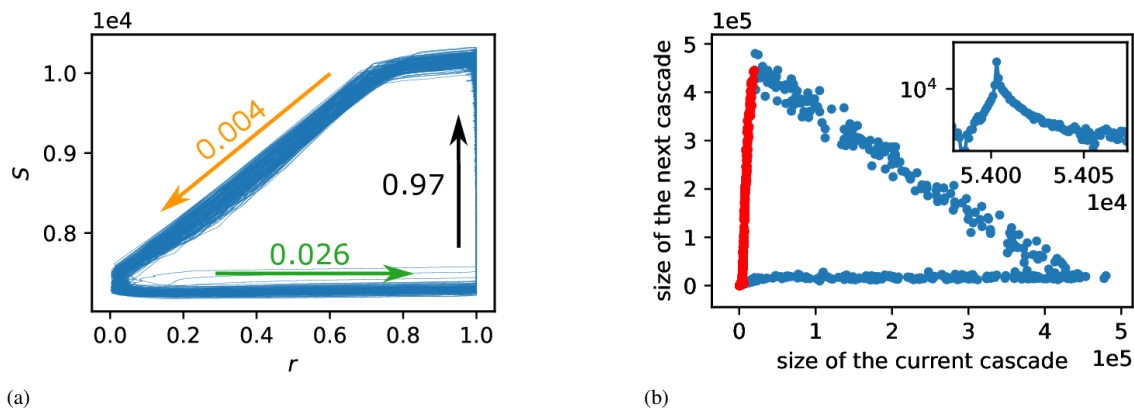


FIG. 1: (a) Total load S vs. global synchronization r . The plot shows BTW-KM cycle with its three phases: DK phase (the orange arrow); synchronization phase (green arrow); and load buildup phase (black arrow). The numbers next to each arrow indicate the fraction of time spent in that phase. (b) Evolution of cascade sizes plotted in the spirit of discrete dynamical systems. Red points correspond to pairs of events after which the synchronization order parameter is still $r > 0.1$ and constitute the DK event. The inset shows time evolution of logarithm of cascade sizes around one DK event.

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