## Empirical social triad statistics can be explained with dyadic homophylic interactions

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The remarkable robustness of many social systems has been associated with a peculiar triangular structure in the underlying social networks. Triples of people that have three positive relations (e.g. friendship) between each other are strongly overrepresented. Triples with two negative (e.g. enmity) and one positive relation are also overrepresented, triples with one or three negative relations are drastically suppressed. For almost a century, the mechanism behind this very specific ("balanced") triad statistics remains elusive. We argue that such triangle statistics can also emerge from *dyadic* interactions if these interactions are determined by homophily between agents. Without knowledge of triads in their neighbourhoods, the agents modify their opinions so as to minimize a social tension defined via the weighted sum of opinion overlaps with friends and opinion discordance with enemies. The model exhibits a transition from unbalanced- to balanced society at a critical temperature which depends on the number of opinions, G, the mean degree, K, and the relative strength of positive interactions to that of negative ones,  $\alpha$ . As  $\alpha$  exceeds 1/2, a transition between the absorbing states with different fractions of balanced triads occurs. We further demonstrate the quality of the model on detailed temporal relation-data of a society of thousands of players of a massive multiplayer online game where we can observe triangle formation directly. It not only successfully predicts the distribution of triangle types but also explains empirical group-size distributions, which are essential for social cohesion.

## References

[1] Empirical social triad statistics can be explained with dyadic homophylic interactions, T. M. Pham, J. Korbel, R. Hanel and S. Thurner, PNAS e2121103119 (2022).



Figure 1: Order parameter,  $f = (n_+ - n_-)/(n_+ + n_-)$ , where  $n_+$  and  $n_-$  are the numbers of balanced and unbalanced triads, respectively, as a function of the relative strength of positive interactions to that of negative ones,  $\alpha$ , and the inverse temperature,  $\beta$ , for two number of opinions: (a) G = 9 and (b) G = 27. Results are averaged over 100 runs with K = 8 and N = 1000.