Local balance index of signed networks

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Ever since Heider's seminal work [1], the notion of structural balance has played a key role in the analysis of alliances and enmities in social networks -from internet users to international powers. In this context, several authors have proposed a variety of indices to measure the level of unbalance of a network, such as the Estrada-Benzi global balance index [2, 3] of a signed graph with adjacency matrix A:

$$K_{gl} = \frac{\mathrm{tr}(e^A)}{\mathrm{tr}(e^{|A|})}. \quad (1)$$

However, all proposed indices measure the balance of the network as a whole, without detailing which nodes are responsible for unbalancing the network. In contrast, we propose an *local balance index*, K_i, defined as:

$$K_i = \frac{(e^A)_{ii}}{(e^{|A|})_{ii}} \quad (2)$$

We show that the index K_i has the bounds $0 < K_i \leq 1$, where the upper bound is reached if and only if the network is balanced and the lower bound is approached when the considered node is part of several unbalanced cycles of short length. Moreover, the average balance index $K_{avg} = \frac{1}{N}\sum_i K_i$ does not generally coincide with global balance index K_{gl}, suggesting some kind of emergent behavior. We identify examples of synthetic and empirical networks where $K_{gl} < K_{loc}$, $K_{gl} = K_{loc}$ and $K_{gl} > K_{loc}$ (fig. 1), and conclude that $K_{gl} < K_{loc}$ if and only if nodes with a large subgraph centrality are highly unbalanced.

Armed with this mathematical framework, we turn our attention to the network of international diplomatic relations between the years 1814 and 2014 and analyze the time series of the local balance index for each country. We find that the sudden drops in the time series can be used to identify not only the major global conflicts of the 19th and 20th centuries, but also systemic instabilities in countries, even in the absence of war. This is the case of the revolutionary wave of 1848, where several European countries suddenly reduce their local balance, even when hostile relations are absent -for example, the local balance of Baden, Saxony and Hannover drops significantly even though these nodes only participate in positive interactions. These findings show the complex, non-local nature of geopolitical conflicts and suggest that the topological structure of diplomatic networks plays a major role in the onset and evolution of international conflicts.

[1] Heider, Fritz. "The Psychology of Interpersonal Relations". John Wiley and Sons (1958).

[2] Estrada, Ernesto, and Michele Benzi. "Walk-based measure of balance in signed networks: Detecting lack of balance in social networks." Physical Review E 90.4 (2014): 042802.

[3] Estrada, Ernesto. "Rethinking structural balance in signed social networks." Discrete Applied Mathematics 268 (2019): 70-90.

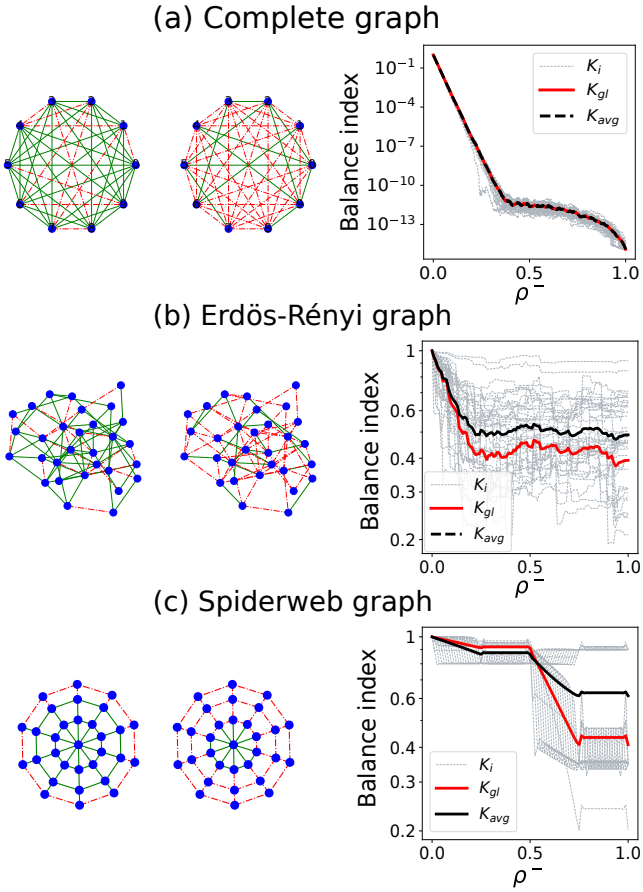

Fig. 1. Local (grey), average (black) and global (red) balance as a function of the fraction of negative edges, ρ^-, for three types of networks: the complete graph, the Erdos-Renyi random network and the spider web graph. We have also plotted two examples of signed networks of each family, with different values of ρ^-. Green edges represent positive interactions and red edges, negative ones.