Stable Chimera States via Geometric Singular Perturbation Theory.

Luis Guillermo Venegas Pineda[1], Hildeberto Jardón-Kojakhmetov[2], and Ming Cao.[3]

[1] *Faculty of Science and Engineering, Centre for Data Science and Systems Complexity, Bernoulli Institute, University of Groningen.* l.g.venegas.pineda@rug.nl

[2] *Faculty of Science and Engineering, Dynamical Systems, Geometry and Mathematical Physics, Bernoulli Institute, University of Groningen.* h.jardon.kojakhmetov@rug.nl

[3] *Faculty of Science and Engineering, Discrete Technology and Production Automation, Engineering and Technology Institute Groningen, University of Groningen.* m.cao@rug.nl

July 28, 2022

Abstract

Over the last decades chimera states have attracted considerable attention as unexpected symmetry-broken spatio-temporal patterns exhibiting simultaneous synchronous and incoherent behaviour under particular conditions. Although relevant results of such unforeseen states in different physical and topological configurations have been obtained, there remain several structures and mechanisms yet to be unveiled. In this presentation, I will address the case of coevolutive coupling occurring in a multilayer network composed by two populations of different kinds of heterogeneous phase oscillators via a mean-field approach. Moreover, a time-scale separation between the dynamics of the elements and the adaptive coupling strength connecting them allows the employment of Geometric Singular Perturbation Theory (GSPT) results in order to obtain a better insight on the dynamics of the system from a fast-slow perspective. At first, I will propose a mechanism for the emergence of stable chimera states through the inclusion of a coevolutive intercoupling strength connecting the populations. In this setting, I will provide with neccesary conditions for the critical manifold to be normally hyperbolic along the entire domain of interest. Moreover, I will discuss the necessary conditions for the critical manifold to be strictly attractive, which will lead to the generation of stable chimera states dependent only on the slow flow dynamics employed. Thereafter, I will compare the aforementioned results with the effects of incorporating a coevolutive intracoupling strength over one of the populations. For this specific scenario, I will provide with the necessary conditions for the critical manifold to exists and be composed by connected sets throughout the domain of interest, as well as obtain neccesary conditions for this manifold to be normally hyperbolic and strictly attractive. Finally, the possibility of producing breathing chimera states by specific coevolutionary coupling dynamics will be discussed and numerical simulations supporting our findings will be presented.

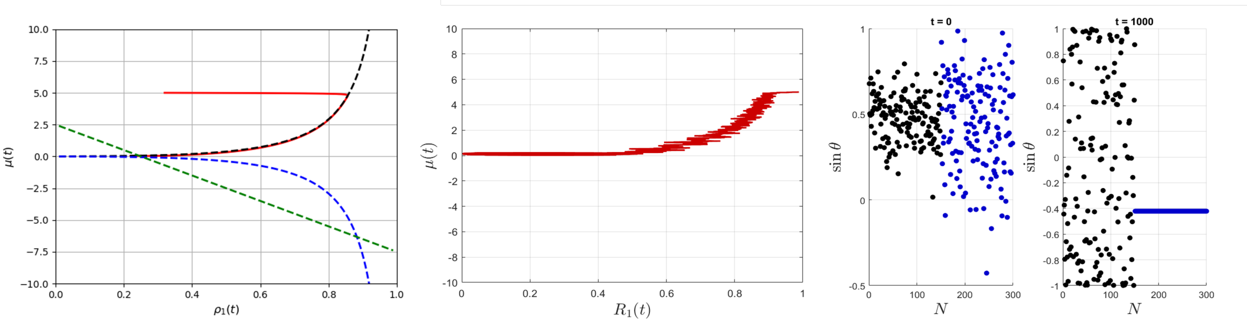

Figure 1: Comparison between mean-field reduction (left), and network system (center) with the same macroscopic quantities. For the mean-field approach, the black and blue lines represent the critical manifold branches while the green line describes the set of equilibrium points for the slow flow. Oscillator phases for the network system (right) at initial ($t = 0$ [s]) and final time ($t = 1000$ [s]) demonstrating the emergence of stable chimera states. Black and blue dots represent the nodes of the first and second population, respectively.