Identifying Limitations in Analytic Models of Percolation on Networks

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The robustness of networks against random node removal is an extensively studied problem, with various theoretical models having been developed which attempt to predict and explain robustness given certain information about network structure. The most simplistic requires only the degree distribution of a network [1], and this has been refined into models that also take into account degree-degree correlations [2] and clustering [3]. However, these models are not necessarily always accurate, and can fail to capture elements of network structure that are indicative of robustness.

First, we demonstrate the limitations of these models by examining the robustness of real-world networks, assessing simulated node removal on real world structures and comparing these results to the predictions given by theoretical models. An example is given in Figure 1. Second, we introduce a new model, which makes predictions based on “local tree likeness”, which is a function of how many second neighbours a given node has. The results from this model are given in Figure 2. This new model clearly outperforms the other pre-existing methods.

![Figure 1: Robustness predictions based on degree distribution compared to real robustness.](image1)

![Figure 2: Robustness predictions based on local tree likeness compared to real robustness.](image2)

Thirdly, we identify the conditions for which all models (including ours) are unable to make good predictions. We find that they consistently overestimate the robustness of networks that are both highly modular and have modules that are far apart from one another, which we refer to as “highly dispersed”. When these highly modular, highly dispersed networks are excluded from our data, we see that the overall quality of our predictions improves significantly, as seen in Figure 3. We go on to consider how accurate robustness predictions for these highly modular, highly dispersed networks may be made in future.

![Figure 3: Robustness predictions based on local tree likeness compared to real robustness without highly modular, highly dispersed networks.](image3)

References